Math 53: Multivariable Calculus **Sections 102, 103**

Worksheet for 2020-01-24

Conceptual questions

Question 1. Determine which of the following polar coordinates (r, θ) does NOT represent the same point as the other four:

$$(3, -5\pi/4), (3, 3\pi/4), (-3, -\pi/4), (-3, 3\pi/4), (-3, 7\pi/4)$$

Question 2. What shape in the xy-plane does the polar curve $r = \csc\theta$ describe?

Question 3. How is the polar curve $r = f(\theta - \pi/4)$ related to the polar curve $r = f(\theta)$? Use this to find a polar equation for the circle centered at $(x, y) = (1, 1)$ passing through the origin (i.e. with radius $\sqrt{2}$).

Question 4. One the backside of this sheet of paper, indicate **all** regions in the given r, θ-grid that correspond to the shaded regions A and B in Figure 1.

Computations

Problem 1. Find the slope of the tangent line to the polar curve $r = 1/\theta$ at the point where $\theta = \pi$.

Problem 2 (Stewart §10.3.64)**.** Find all points on the polar curve $r = e^{\theta}$ where the tangent line is either horizontal or vertical.

Problem 3. Find a polar equation $r = f(\theta)$ for the circle centered at the point (a, b) (given in Cartesian coordinates) passing through the origin[1].

What θ interval traces out the circle once? For the particular case $a = 1, b = 1$, how does your answer for this problem compare to your answer for Question 3 above?

Problem 4. This problem is about the polar curve $r = 2 + \cos(3\theta/2)$ graphed in Figure 2. This curve also appears in Stewart §10.3.54 (a matching problem).

(a) Find the Cartesian coordinates of the three points of self-intersection.
(b) At the self-intersection point in the first quadrant, there are *two* tangent lines. Find their equations.

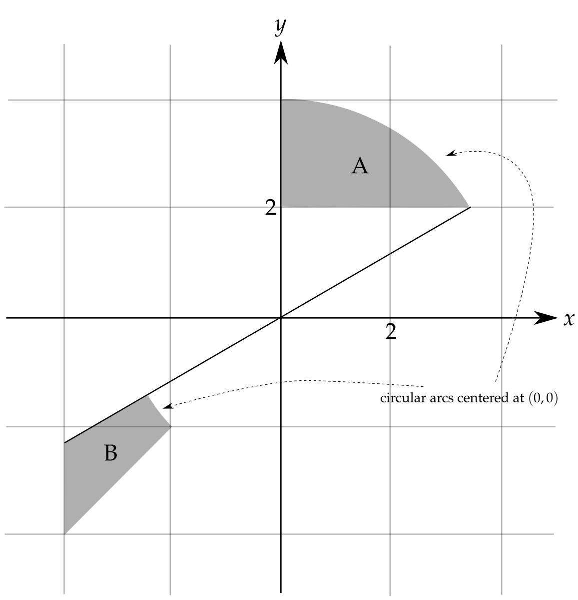


FIGURE 1. The two curved arcs are parts of circles centered at the origin. All other sides are straight lines.

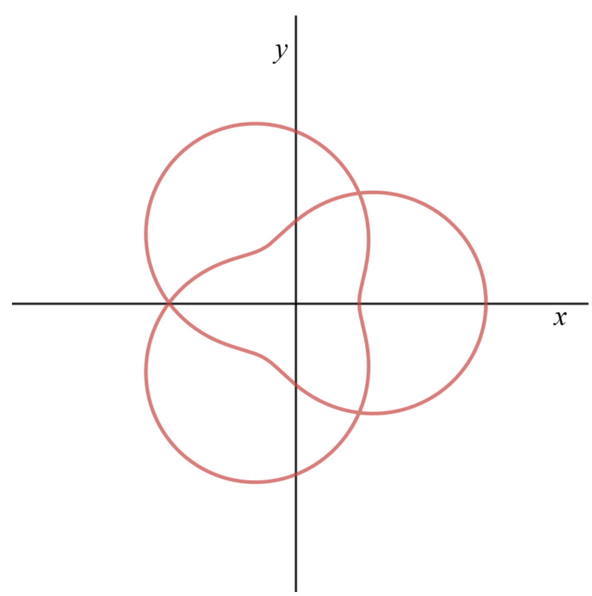


FIGURE 2. The polar curve $r = 2 + \cos(3\theta/2)$.

[1] The only circles which have a "nice" polar form are those centered at the origin or passing through the origin.

Below are the numerical answers to the worksheet exercises. If you would like a more detailed solution, feel free to ask me in person. (Do let me know if you catch any mistakes!)

For this worksheet, the computational problems will be recycled, so their answers have been omitted from the below.

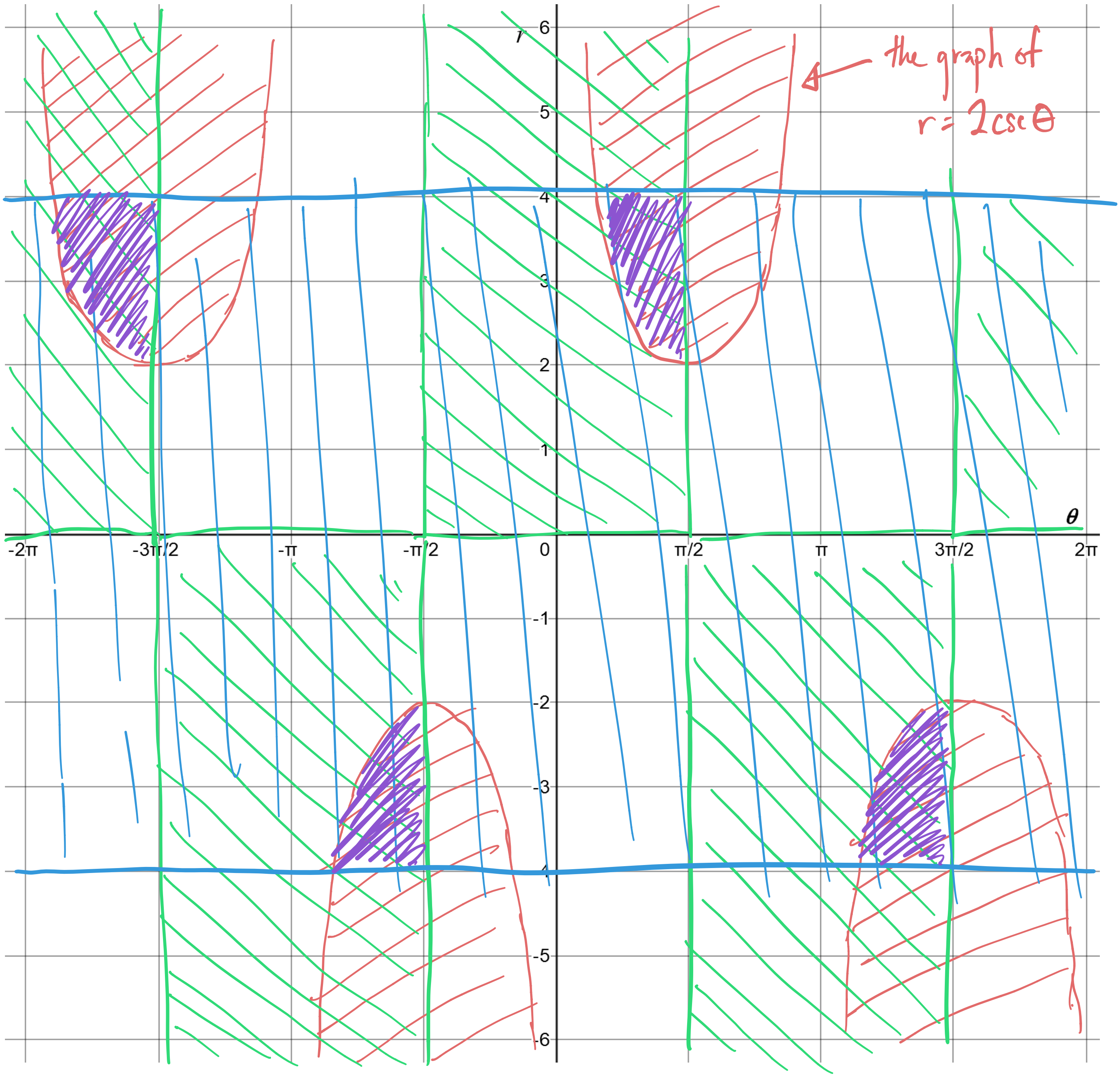
Answers to conceptual questions

Question 1. $(-3, 3\pi/4)$.

Question 2. Horizontal line $y = 1$.

Question 3. The graph of $r = f(\theta - \pi/4)$ in the xy-plane is that of $r = f(\theta)$ rotated counterclockwise by $\pi/4$ radians. The polar equation in question is $r = 2\sqrt{2}\cos(\theta - \pi/4)$.

Question 4. A picture is included on the following page.

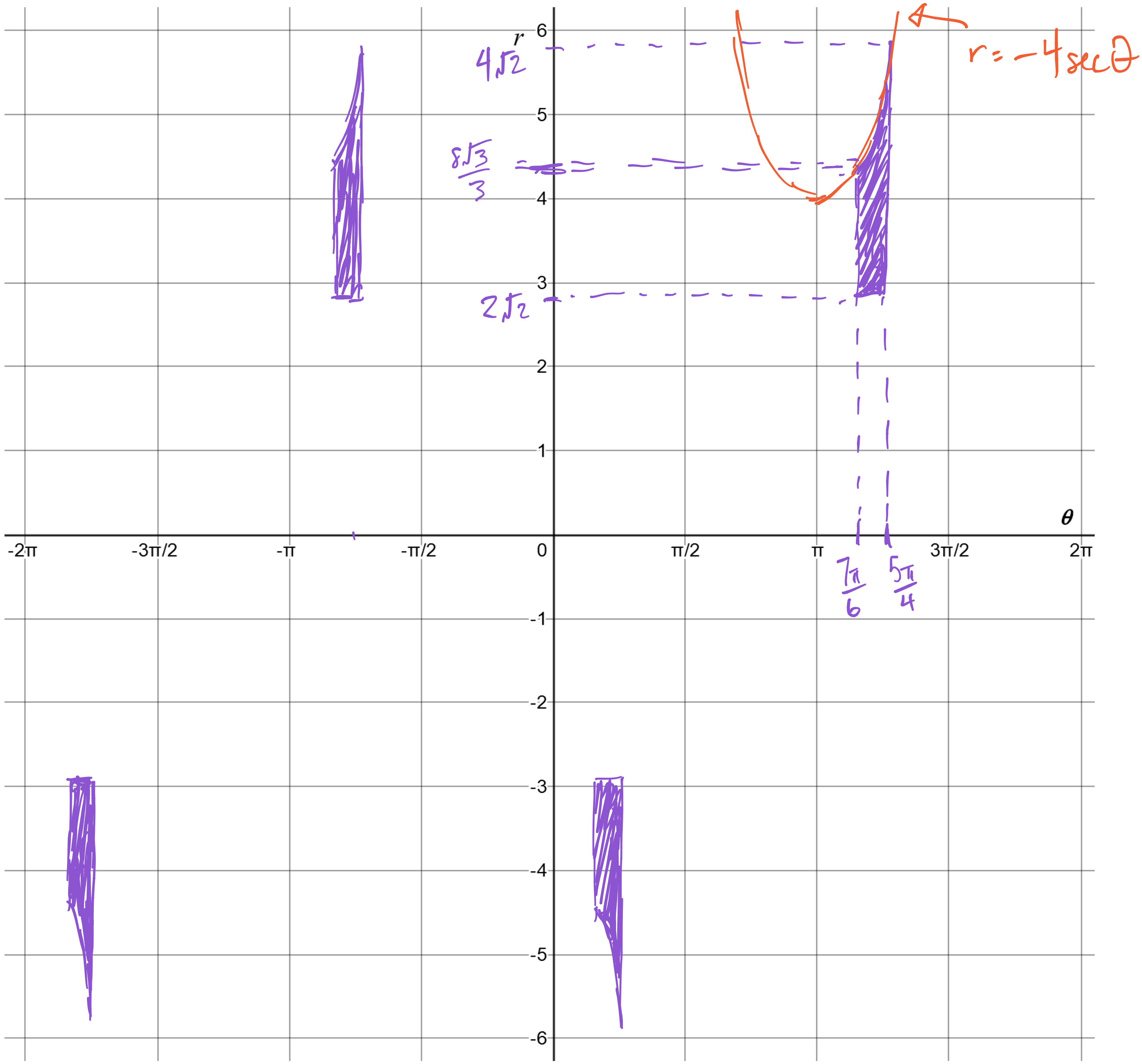


Region A is defined by

$y \geq 2$	i.e.	$r\sin\theta > 2$
$x \geq 0$		$r\cos\theta \geq 0$
$x^2+y^2 \leq 16$		$\lvert r \rvert \leq 4$

The regions satisfying all three constraints has been shaded in purple.

($\geq$ vs $>$ is not important in this problem)



I'll let you check that this is the answer for region B.